Problem (1) Consider the following proposed rule for functional dependencies: If \( \alpha \rightarrow \beta \) and \( \gamma \rightarrow \beta \), then \( \alpha \rightarrow \gamma \). Prove that this rule is not sound by showing a relation \( r \) that satisfies \( \alpha \rightarrow \beta \) and \( \gamma \rightarrow \beta \), but does not satisfy \( \alpha \rightarrow \gamma \).

Problem (2) Suppose that we are given the schema \( R = (A, B, C, D, E) \) and the following set \( F \) of functional dependencies holds

\[
\begin{align*}
A &\rightarrow BC \\
CD &\rightarrow E \\
B &\rightarrow D \\
E &\rightarrow A
\end{align*}
\]

a) Please compute \( B \)'s closure \( B^+ \).

b) Please identify all candidate keys of the relation \( R \) using attribute closure.

c) What is the highest norm form of relation \( R \)? Please explain why.

d) Show that the following decomposition of \( R \) is a lossless decomposition

\[
(A, B, C) \\
(A, D, E)
\]

e) Show the following decomposition of \( R \) is not a lossless decomposition

\[
(A, B, C) \\
(C, D, E)
\]