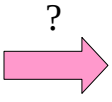
Association Rule Mining

Generating assoc. rules from frequent itemsets

- Assume that we have discovered the frequent itemsets and their support
- How do we generate association rules?
- Frequent itemsets:

{1}	2
{2}	3
{3}	3
{5}	3
{1,3}	2
{2,3}	2
{2,5}	3
{3,5}	2
{2,3,5}	2



For each frequent itemset *l* find all nonempty subsets *s*. For each *s* generate rule $s \Rightarrow l\text{-}s$ if sup(l)/sup(s)≥min_conf

Example: for {2,3,5}, min_conf = 75%

{2,3} ⇒ 5 √
{2,5} ⇒ 3 **X**
{3,5} ⇒ 2 √

Discovering Rules

- Naïve Algorithm

```
for each frequent itemset l do
  for each subset c of l do
    if (support(l ) / support(l - c) >= minconf) then
      output the rule (l – c ) ⇒ c,
        with confidence = support(l ) / support (l - c )
         and support = support(l )
```

Discovering Rules (2)

- **Lemma**. If consequent c generates a valid rule, so do all subsets of c. (e.g. X ⇒ YZ, then XY ⇒ Z and XZ ⇒ Y)
- Example: Consider a frequent itemset ABCDE

If ACDE ⇒ B and ABCE ⇒ D are the only one-consequent rules with minimum support confidence, then

ACE ⇒ BD is the only other rule that needs to be tested

Is Apriori Fast Enough? — Performance Bottlenecks

- The core of the Apriori algorithm:
  - Use frequent (k – 1)-itemsets to generate candidate frequent k-itemsets
  - Use database scan and pattern matching to collect counts for the candidate itemsets
- The bottleneck of *Apriori*: candidate generation
  - Huge candidate sets:
    - 10^4 frequent 1-itemset will generate 10^7 candidate 2-itemsets
    - To discover a frequent pattern of size 100, e.g., $\{a_1, a_2, \ldots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ candidates.
  - Multiple scans of database:
    - Needs (n +1) scans, n is the length of the longest pattern

FP-growth: Mining Frequent Patterns Without Candidate Generation

- Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- Develop an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - Avoid candidate generation: sub-database test only!

FP-tree Construction from a Transactional DB

TID	Items bought	(ordered) frequent items
100	{f, a, c, d, g, i, m, p}	{f, c, a, m, p}
200	{a, b, c, f, l, m, o}	{f, c, a, b, m}
300	{b, f, h, j, o}	{f, b}
400	{b, c, k, s, p}	{c, b, p}
500	{a, f, c, e, l, p, m, n}	{f, c, a, m, p}

min_support = 3

Item	*frequency*
f	*4*
c	*4*
a	*3*
b	*3*
m	*3*
p	*3*

Steps:

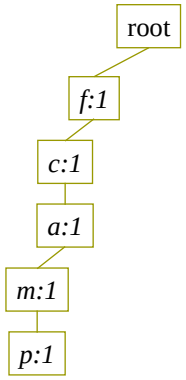
1. Scan DB once, find frequent 1-itemsets (single item patterns)
2. Order frequent items in descending order of their frequency
3. Scan DB again, construct FP-tree

FP-tree Construction

min_support = 3

Item	*frequency*
f	*4*
c	*4*
a	*3*
b	*3*
m	*3*
p	*3*

TID	freq. Items bought
~~100~~	~~{f, c, a, m, p}~~
200	{f, c, a, b, m}
300	{f, b}
400	{c, p, b}
500	{f, c, a, m, p}



FP-tree Construction

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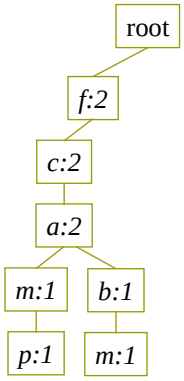

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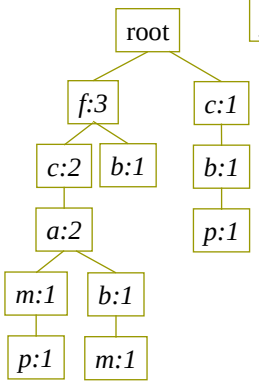


FP-tree Construction

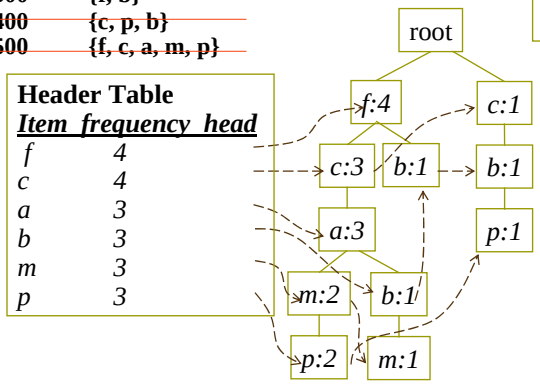
min_support = 3

Item	*frequency*
f	*4*
c	*4*
a	*3*
b	*3*
m	*3*
p	*3*

TID	freq. Items bought
100	{f, c, a, m, p}
200	{f, c, a, b, m}
300	{f, b}
400	{c, p, b}
500	{f, c, a, m, p}

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



Benefits of the FP-tree Structure

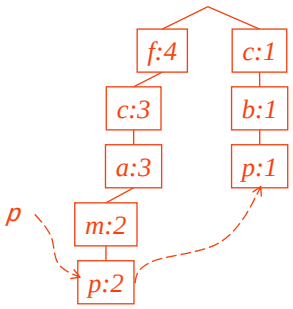
- Completeness:
  - never breaks a long pattern of any transaction
  - preserves complete information for frequent pattern mining
- Compactness
  - reduce irrelevant information—infrequent items are gone
  - frequency descending ordering: more frequent items are more likely to be shared
  - never be larger than the original database (if not count node-links and counts)
  - Example: For Connect-4 DB, compression ratio could be over 100

Mining Frequent Patterns Using FP-tree

- General idea (divide-and-conquer)
  - Recursively grow frequent pattern path using the FP-tree
- Method
  - For each item, construct its conditional pattern-base, and then its conditional FP-tree
  - Repeat the process on each newly created conditional FP-tree
  - Until the resulting FP-tree is empty, or it contains only one path (single path will generate all the combinations of its sub-paths, each of which is a frequent pattern)

Mining Frequent Patterns Using the FP-tree (cont'd)

- Start with last item in order (i.e., p).
- Follow node pointers and traverse only the paths containing p.
- Accumulate all of transformed prefix paths of that item to form a *conditional pattern base*



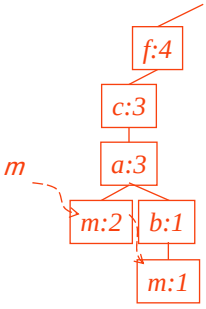
***Conditional* pattern base for p**

fcam:2, cb:1

Construct a new FP-tree based on this pattern, by merging all paths and keeping nodes that appear ≥sup times. This leads to only one branch *c:3*
Thus we derive only one frequent pattern cont. *p*. Pattern *cp*

Mining Frequent Patterns Using the FP-tree (cont'd)

- Move to next least frequent item in order, i.e., *m*
- Follow node pointers and traverse only the paths containing *m*.
- Accumulate all of transformed prefix paths of that item to form a *conditional pattern base*



***m*-conditional pattern base:**
fca:2, fcab:1

{}
f:3
c:3
a:3

All frequent patterns that include *m*

***m*,**
fm, cm, am,
fcm, fam, cam,
fcam

***m*-conditional FP-tree (contains only path *fca:3*)**

Properties of FP-tree for Conditional Pattern Base Construction

- Node-link property
  - For any frequent item a_i, all the possible frequent patterns that contain a_i can be obtained by following a_i's node-links, starting from a_i's head in the FP-tree header
- Prefix path property
  - To calculate the frequent patterns for a node a_i in a path P, only the prefix sub-path of a_i in P need to be accumulated, and its frequency count should carry the same count as node a_i.

Conditional Pattern-Bases for the example

Item	Conditional pattern-base	Conditional FP-tree
p	{(fcam:2), (cb:1)}	{(c:3)}|p
m	{(fca:2), (fcab:1)}	{(f:3, c:3, a:3)}|m
b	{(fca:1), (f:1), (c:1)}	Empty
a	{(fc:3)}	{(f:3, c:3)}|a
c	{(f:3)}	{(f:3)}|c
f	Empty	Empty

Principles of Frequent Pattern Growth

- Pattern growth property
  - Let α be a frequent itemset in DB, B be α's conditional pattern base, and β be an itemset in B. Then $\alpha \cup \beta$ is a frequent itemset in DB iff β is frequent in B.
- "*abcdef* " is a frequent pattern, if and only if
  - "*abcde* " is a frequent pattern, and
  - "*f* " is frequent in the set of transactions containing "*abcde* "

Why Is Frequent Pattern Growth Fast?

- Performance studies show
  - FP-growth is an order of magnitude faster than Apriori, and is also faster than tree-projection
- Reasoning
  - No candidate generation, no candidate test
  - Uses compact data structure
  - Eliminates repeated database scan
  - Basic operation is counting and FP-tree building

FP-growth vs. Apriori: Scalability With the Support Threshold