Problem 1

a)

$$B^+ = \{B\}\ (Start)$$

$$B^+ = \{B, D\}\ (B \rightarrow D)$$

b)

$$A^+ = \{A\}\ (Start)$$

$$A^+ = \{A, B, C\}\ (A \rightarrow BC)$$

$$A^+ = \{A, B, C, D\}\ (B \rightarrow D)$$

$$A^+ = \{A, B, C, D, E\}\ (CD \rightarrow E)$$

A is a candidate key.

$$B^+ = \{B, D\}\ (From\ part\ a)$$

B is not a candidate key.

$$C^+ = \{C\}\ (Start)$$

C is not a candidate key.

$$D^+ = \{D\}\ (Start)$$

D is not a candidate key.

$$E^+ = \{E\}\ (Start)$$

$$E^+ = \{A, E\}\ (E \rightarrow A)$$

$$E^+ = \{A, B, C, E\}\ (A \rightarrow BC)$$

$$E^+ = \{A, B, C, D, E\}\ (B \rightarrow D)$$

E is a candidate key.

$$\{B, C\}^+ = \{B, C\}\ (Start)$$

$$\{B, C\}^+ = \{B, C, D\}\ (B \rightarrow D)$$

$$\{B,C\}^+ = \{B,C,D,E\}\ (CD \rightarrow E)$$

$$\{B,C\}^+ = \{A,B,C,D,E\}\ (E \rightarrow A)$$

{B,C} is a candidate key.

$$\{C,D\}^+ = \{C,D\}\ (Start)$$

$$\{C,D\}^+ = \{C,D,E\}\ (CD \rightarrow E)$$

$$\{C,D\}^+ = \{A,C,D,E\}\ (E \rightarrow A)$$

$$\{C,D\}^+ = \{A,B,C,D,E\}\ (A \rightarrow BC)$$

{C,D} is a candidate key.

Our candidate keys are A, E, {B,C}, and {C,D}.

c)

We know that the relation is 1NF under all candidate keys because there are no multivalued attributes.

We also know that the relation is 2NF because there are no non-key attributes that are partially dependent on a set of key attributes.

We also know that the relation is 3NF because there are no non-key attributes that depend on other non-key attributes.

However, the relation is not BCNF because B alone is not a candidate key and the function dependency B->D exists.

The highest normal form is 3NF.

d)

The decomposition can be verified as lossless using the algorithm described on pages 554 and 555 of the textbook. The matrix S is initialized to this:

	A	B	C	D	E
R1	a_0	a_1	a_2	b_{03}	b_{04}
R2	a_0	b_{11}	b_{12}	a_3	a_4

After the first iteration of the loop in step 4, the matrix will look like this:

	A	B	C	D	E
R1	a_0	a_1	a_2	b_{03}	b_{04}
R2	a_0	a_1	a_2	a_3	a_4

Since R2 is made up entirely of ‘a’ symbols, the decomposition has the nonadditive join property, meaning the decomposition is lossless.

e)

The decomposition can be verified as not lossless using the algorithm described on pages 554 and 555 of the textbook. The matrix S is initialized to this:

	A	B	C	D	E
R1	a_0	a_1	a_2	b_{03}	b_{04}
R2	b_{10}	b_{11}	a_2	a_3	a_4

After one complete loop execution, the matrix will remain unchanged. Also, there are no rows that consist of only ‘a’ elements. Because of this, the decomposition does not have the nonadditive join property, meaning the decomposition is lossy.

Problem 2

The following sets of attributes can form keys of R:

{Course_no, Sec_no, Semester, Year}

{Room_no, Days_hours, Semester, Year}

There is some redundancy – for each semester and each section a course is offered, the offering department, credit hours, and course level are unnecessarily reiterated. The relation can be normalized as follows:

R1 = {Course_no, Offering_dept, Credit_hours, Course_level}

R2 = {Course_no, Sec_no, Semester, Year, Days_hours, Room_no, No_of_students, Instructor_ssn}

Of course, this decomposition satisfies 1NF since there are no multivalued attributes. This decomposition also satisfies 2NF since there are no non-key attributes that are partially dependent on attributes that are part of a key. The decomposition also satisfies 3NF since all determinants of non-key attributes are candidate keys. Finally, the decomposition is also BCNF since all determinants are candidate keys.

Problem 3

The relation is 1NF since there are no multivalued attributes. However, the relation is not 2NF because Commission% is partially dependent on Salesperson#. Due to this, the relation can be decomposed as follows:

R1 = {Salesperson#, Commission%}

R2 = {Car#, Date_sold, Salesperson#, Discount_amt}

This decomposed relation is 2NF. However, this decomposed relation is not 3NF because the determinant of Disccount_amt, a non-key attribute, is dependent on Date_sold, which is also a non-key attribute. The relation can be further decomposed as shown below:

R1 = {Salesperson#, Commission%}

R2 = {Car#, Date_sold, Salesperson#}

R3 = {Date_sold, Discount_amt}

This decomposed relation is 3NF. It is also BNF since there are no non-key attributes that are determinants.

Problem 4

In this problem, we are assuming that each day a patient visits a doctor, a different treatment will be performed, yet the doctor's diagnosis of the patient is kept constant across all visits. Based on the description of what occurs within a visit to the doctor, the following functional dependencies can be derived:

{Doctor#, Patient#} -> Diagnosis

{Doctor#, Patient#, Date} -> Treat_code

Treat_code -> Charge

Due to this, the only candidate key for this relation is {Doctor#, Patient#, Date}.

This relation is not 2NF. This is because there Diagnosis is partially dependent on {Doctor#, Patient#}. Due to this, the relation can be decomposed as follows to achieve 2NF:

R1: {Doctor#, Patient#, Date, Treat_code, Charge}

R2: {Doctor#, Patient#, Diagnosis}

The above relation is 2NF. However, it is not 3NF because Charge, a non-key is dependent on Treat_code, which is also a non-key. The relation can be further decomposed as follows:

R1: {Doctor#, Patient#, Date, Treat_code}

R2: {Doctor#, Patient#, Diagnosis}

R3: {Treat_code, Charge}