Optimization Problems and Wrap-Up

CS 221 Lecture 14

Tue 6 December 2011

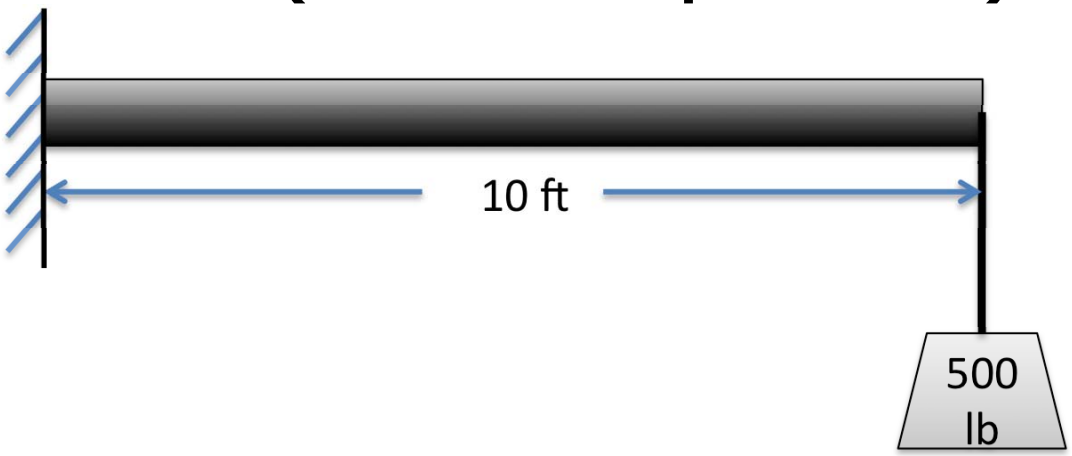
Agenda

1. Announcements
2. Solving Optimization Problems in Excel and MATLAB (Text Chapter 10)
3. Other nifty functions in (standard) MATLAB
   - Image processing
   - Audio processing
4. Summary: What we've learned
5. Course Evaluation Survey

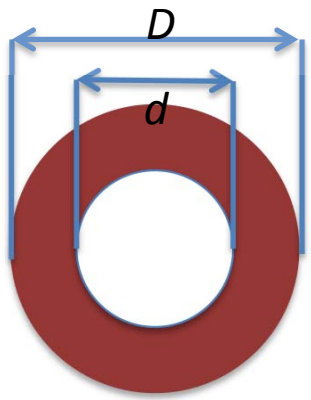
1. Announcements

- Thursday is catch-up and get help day in Lab
- Final Exam is Thursday 15 Dec 10:30-12:30
  - Location: here
- Homework 5 is due; solutions will be posted this weekend
- Extra Credit problems are available
  - Due Sunday 11 December
  - Two problems, worth a total of 5% of your grade!

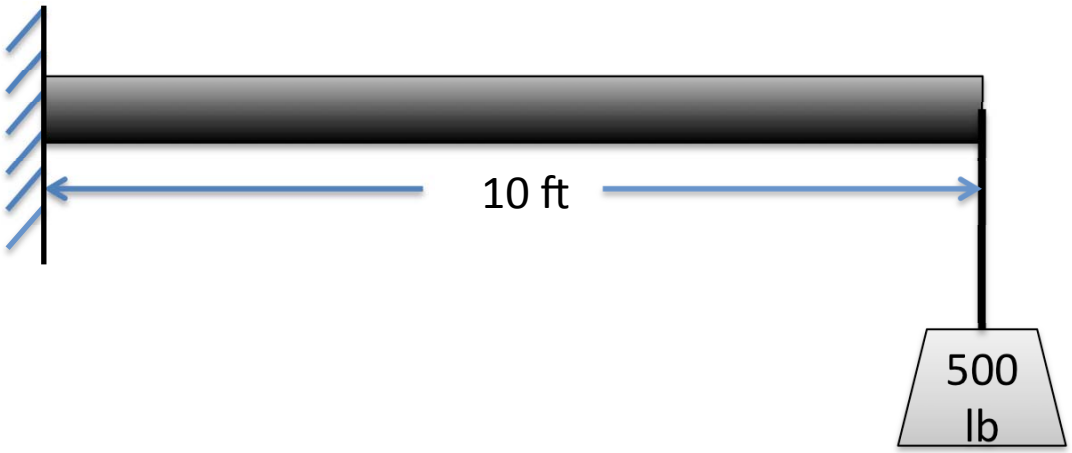
2. Solving Optimization Problems (Text Chapter 10)



- Engineer must specify a pipe, 10′ long, to hold a 500-lb weight with at most 1″ deflection
- Two design variables:
  - inside diameter *d*
  - outside diameter *D*

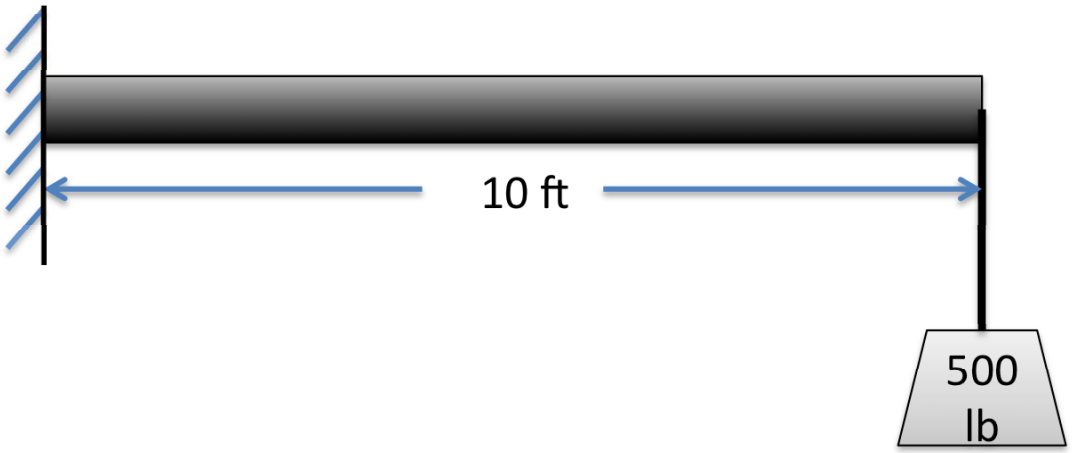


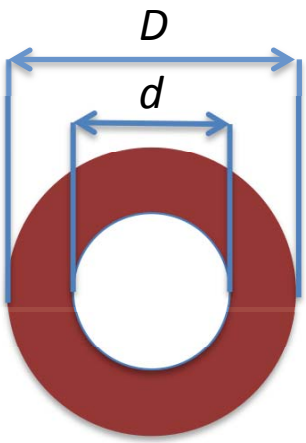
How to Solve It?



- Deflection δ is given by: $\delta = 65W\,\ell^3 / 3\pi E(D^4 - d^4)$
  - W = weight (500 lb)
  - ℓ = length (10 feet = 120 inches)
  - E = modulus of elasticity (for steel: 3×10^7 lb/sq.in)
- Require: $\delta \leq 1''$

How to Solve It?



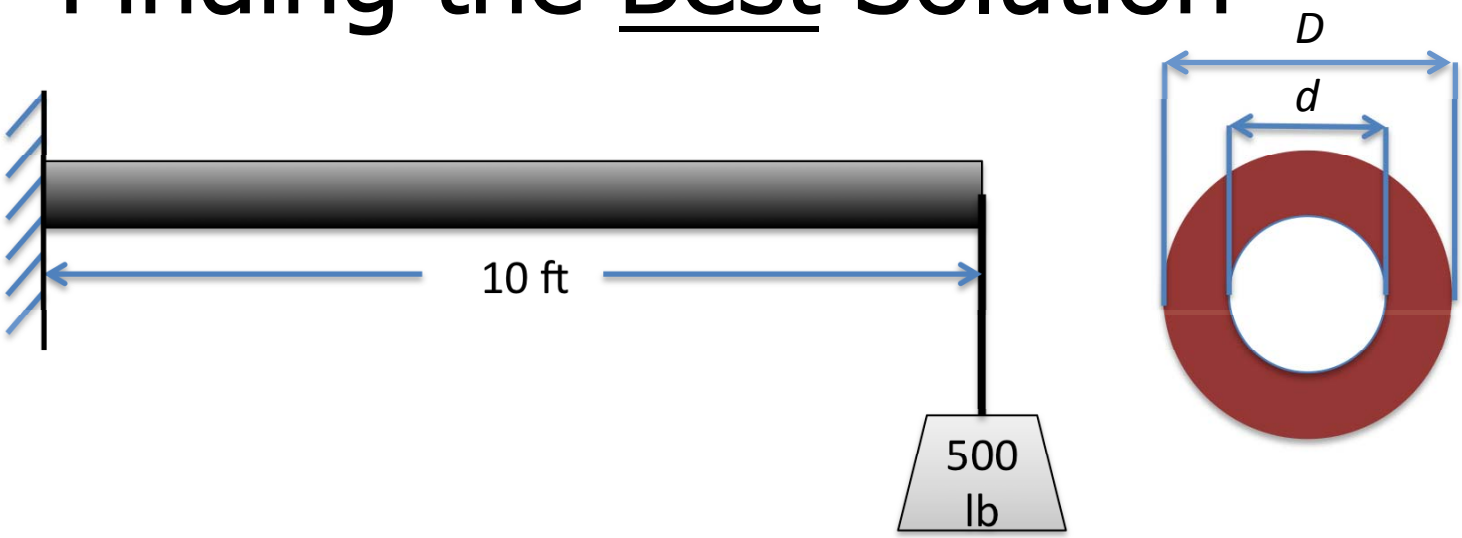


Straightforward: just pick a value for one of the design variables, set $\delta = 1$, solve for the other

- Setting D = 4″ gives d = 2.79″ (Thickness: 1.2″)
- Setting d = 1″ gives D = 3.74″ (Thickness: 2.74″)

But…

Finding the Best Solution



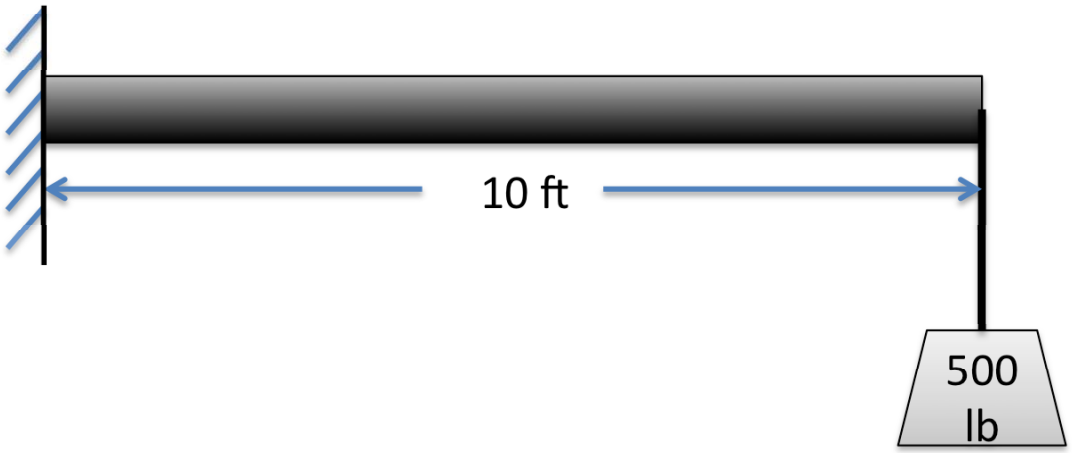
As Engineers, we want to get the best solution.

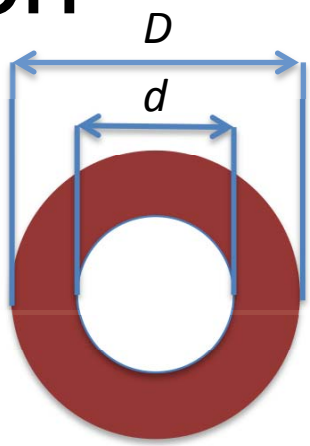
- By picking one of the values, we might miss something

In this case: we probably want to minimize cost of the pipe. Cost is probably proportional to the volume of steel in the pipe, which is given by:

$$V = l\pi(D^2 - d^2)/4$$

Finding the Best Solution





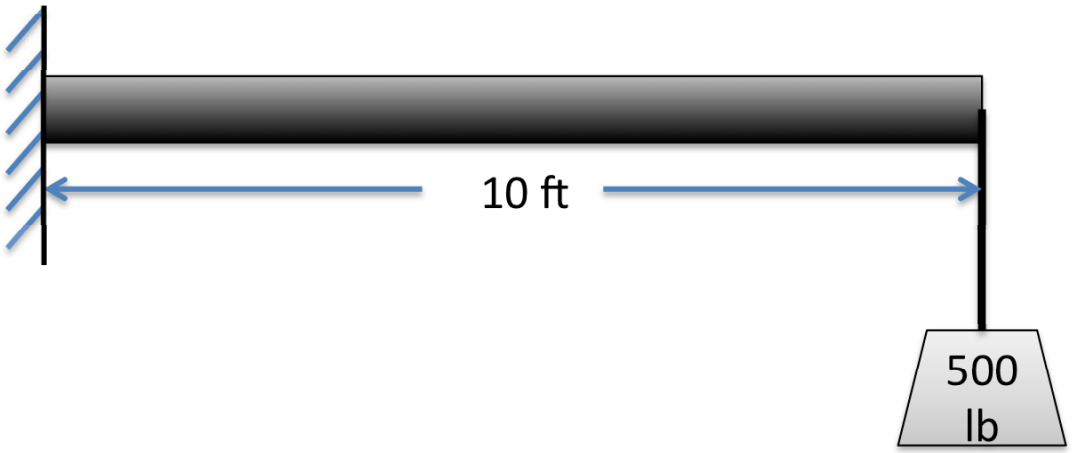
So: we want to pick D and d so as to minimize

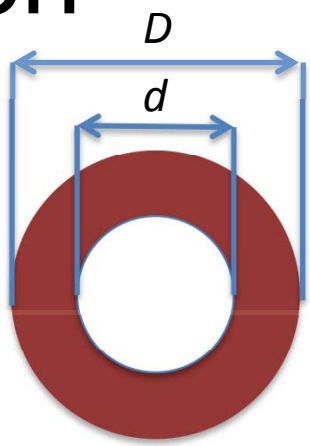
$$V = l\pi(D^2 - d^2)/4$$

while also satisfying

$$65Wl^3 / 3\pi E(D^4 - d^4) \leq 1$$

Finding the Best Solution





Other constraints:

- D and d can't be negative
- The pipe probably needs to have some minimum wall thickness to be manufacturable (say 0.125")

The Final Problem

Choose D and d to

Minimize $V = l\pi(D^2 - d^2)/4$ } Objective Function

Subject to:

- $65Wl^3 / 3\pi E(D^4 - d^4) \leq 1$
- $D \geq 0$
- $d \geq 0$
- $(D - d)/2 \geq 0.125$

} Constraints

This is a nonlinear, constrained, multivariable optimization problem

Standard Form for Optimization Problems

- Minimize/Maximize $\boxed{f(x_1,x_2,x_3,x_4,\ldots,x_k)}$ Objective Function
- Subject to:
  - $g_1(x_1,\ldots,x_k) = C_1$
  - $g_2(x_1,\ldots,x_k) \geq C_2$

    …
  - $g_N(x_1,\ldots,x_k) \leq C_N$

Design Variables

Constraints – may be equations or inequalities

Classes of Optimization Problems

- Single-/Multi-variable
  - One design variable: simply find maximum/minimum of the objective function
  - Warning: Global maximum/minimum may not exist
- Linear/Nonlinear
  - Depends on the form of the objective function
- Constrained/Unconstrained
  - Constraints may make the problem harder or easier
  - Constraining design variables' values to be integers makes the problem (computationally harder)

Solving Optimization Problems in MATLAB

- fminbnd(): [quasi] constrained, nonlinear, single-variable
  - fminbnd(@objfun,lower,upper)
    - Finds x such that

      objfun(x) is (local) minimum

      lower ≤ x ≤ upper
  - Warning: local vs. global minima
  - Limits on number of iterations
- fminsearch(): unconstrained, nonlinear, multi-variable
  - fminsearch(@objfun,guess)
    - objfun() takes a vector of arguments
    - guess is an "starting point" vector

function handle

function handle

Solving Optimization Problems in Excel

- Unconstrained, nonlinear, single-variable:
  - Use Goal-Seek
- Everything else:
  - Use Solver
- Note: Solver does not come installed by default!
  - Download/install free from Frontline Systems:

    www.solver.com

Summary: Tools for Optimization Problems

- MATLAB:
  - fminsearch() for unconstrained nonlinear multivariable
  - fminbnd() for constrained nonlinear single variable
  - Other tools are in the "Optimization Toolbox" (extra $)
- Excel
  - Goal Seek for unconstrained nonlinear single variable
  - "Solver" for constrained nonlinear multivariable

3. Other MATLAB Capabilities

Image Processing:

- Images are represented as 2-D arrays of pixel values
  - pixel = "picture element"
  - Number of pixels in the array varies
    - E.g., with camera resolution
    - Example image: 2592 x 3888 (= 10077696)
- C = imread('myphoto.jpg')
  - Reads in a JPEG file, returns three-dimensional array C
    - RGB ("Red-Green-Blue" intensity) format
    - Dimensions: image rows, image columns, colors
  - So Red = C(:,:,1); Green = C(:,:,2); Blue = C(:,:,3)
- image(C) displays the photo

Image Processing:

- You can manipulate the array values to play with the image
- Warning: imread() will return color array elements as unsigned 8-bit values (uint8)
  - Maximum value of any element in array: 255
  - Be careful of overflow when doing arithmetic on values
    - Example: averaging pixels in a region (for blurring)
    - Add values to be added in a uint32, then divide.

Audio Files

- load “handel.mat”
  - Reads in variables y (sound wave), Fs (sampling rate) from demo file
- p = audioplayer(y,Fs);
  - Creates an “audioplayer” object that can be used various ways
- play(p);
  - Plays the file
- You can plot, manipulate the waveform (y)...

4. Summary of Things We've Learned

- How to use Excel and MATLAB to perform calculations and solve problems
  - Excel Fundamentals: formatting, built-in functions, formulas
    - Conditionals (IF)
    - Formula updating
  - MATLAB Fundamentals: variables, scripts, built-in functions, vectors and arrays
    - input() – read from keyboard
- Boolean logic: AND, OR, NOT (Truth Tables)
- MATLAB Programming
  - Assignment statements
  - Conditional ("logical") statements: if, if-else, if-elseif-else
  - Iteration statements: while and for
  - User-defined functions: passing parameters, return values

4. Summary of What We've Learned

- MATLAB Programming:
  - Formatting output with fprintf()
  - Nested loops
- Plotting/Graphing with Excel and MATLAB
- Applications:
  - Finding Roots:
    - MATLAB fzero and roots, Excel Goal-Seek
    - Algorithms: bisection,
  - Matrix Operations: product, inversion
  - Solving Systems of Simultaneous Equations
  - Curve-Fitting: fitting models to data (not in text!)
  - Numerical Integration
  - Optimization Problems

Thank you for your participation!