The examination will be closed book and closed notes.

No calculators are allowed. Show your work. You need not calculate numeric answers for large expressions, but you should be able to add and subtract numbers correctly.

In the exam tomorrow you will write your answers on the test paper. (No blue books needed.)

Problem 1. [10 points] A group of 8 students who were studying for finals went to the corner store for midnight snacks; every student bought at least one item. Three students bought energy drinks (and possibly other items). Two bought both energy drinks and frozen burritos (and possibly other items). One bought both ice cream and an energy drink. One bought a frozen burrito and an ice cream treat (and possibly other items). If one student bought all three items, and five students bought ice cream (with other things), how many students bought only a frozen burrito?

Problem 2. [10 points]

a. How many different ways are there to arrange the letters in the word MATHEMATICS?

b. Consider the graph $K_7$. How many cycles of length 6 does it contain? (Recall that when counting cycles we consider rotations and reversals of a cycle to be the same as the original.)

Problem 3. [10 points] Solve the following recurrence relation. (The final answer should not involve complex numbers.)

$$a_n = 5a_{n-1} + 6a_{n-2}$$

where $a_0 = 1$ and $a_1 = 3$. What is $F(5)$?
Problem 4. [20 points]

a. Draw two graphs with six vertices, one that has an Euler trail but not a circuit, and one that has neither an Euler trail nor an Euler circuit.

b. In the space below, draw two graphs of five vertices, one that has a Hamilton cycle and one that does not.

c. Consider a bipartite graph \( G = (V, E) \) with \( V = V_1 \cup V_2, \ |V_1| = 4 \) and \( |V_2| = 3 \). Suppose \( |E| = 12 \). Is \( G \) planar? Justify your answer.

d. Consider the graphs \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \), where \( V_1 = \{1, 2, 3, 4, 5\} \), \( E_1 = \{\{1, 2\}, \{2, 3\}, \{2, 4\}, \{3, 5\}\} \) and \( V_2 = \{a, b, c, d, e\} \), \( E_1 = \{\{a, d\}, \{d, c\}, \{c, b\}, \{b, e\}\} \). Are these graphs isomorphic? Explain your answer.

Problem 5. [10 points] Consider the boolean function \( f(a, b, c) = \prod M(0, 2, 3, 4) \).

a. Write down the Disjunctive Normal Form representation for this function.

b. Explain what a Karnaugh map is used for.

Problem 6. [10 points]

a. Loop-free undirected graph \( G = (V, E) \) has \( \kappa = 1 \) and \( |V| = 10 \). What is the largest number of edges it can have if it is a tree?

b. Given the following graph:

![Graph](image)

Draw the subgraph induced by \( U = \{a, b, c, e\} \).

Problem 7. [10 points] Prove using mathematical induction that for any \( n \geq 5 \), \( n \) can be represented as a sum of integer multiples of two and five (only).

Problem 8. [10 points] Let \( R \) be a relation on \( A \), that is \( R \subseteq A \times A \), where \( |A| = 4 \), and \( |R| = \infty \), that is, \( R \) contains 13 pairs \( (a_1, a_2) \). Consider the following statement:

\[ \exists a \in A \forall a' \in A [(a, a') \in R] \]

Is this statement true or false? If you think it is true, prove it. If you believe it is false, give a counterexample.

Problem 9. [10 points] Write down the following logical laws using only symbols (no words).

a. Conjunction distributes over disjunction.

b. The operator \( \oplus \) is associative.

c. The element \( a \) is the identity of operator \( * \).